

$$\text{Solid} \subset \text{Cond}(Ab)$$

↙
solidification $(-)^{\square}$

$$\forall S = \varinjlim_i S_i \in \text{Prof} \quad \mathbb{Z}[S]^{\square} := \varinjlim \mathbb{Z}[S_i]$$

$$\text{Solid} = \{C \in \text{Cond}(Ab) \mid \text{Hom}(\mathbb{Z}[S]^{\square}, C) \xrightarrow{\sim} \text{Hom}(\mathbb{Z}[S], C)\}$$

Condensed rings : ring = unital assoc. ring

Def A pre-analytic ring A is a triple

$$A = (\underline{A}, A[-], \tau) \quad \text{s.t.}$$

- $\underline{A} \in \text{Cond}(\text{Ring})$

- $A[-] : \text{ED} \rightarrow \text{Mod}_{\underline{A}}^{\text{cond}} = \{ \underline{A}\text{-modules in Cond}(Ab) \}$

$$A[S \amalg S_2] = A[S_1] \times A[S_2]$$

- $\tau : S \rightarrow A[S]$ nat. trans.

Examples

1) $\mathbb{Z}_{\square} = (\underline{\mathbb{Z}}_{\square} = \mathbb{Z}, \mathbb{Z}_{\square}[S] = \mathbb{Z}[S]^{\square}, S \rightarrow \mathbb{Z}[S]^{\square})$

2) $\mathbb{Z} \rightarrow \mathbb{Z}_{\square} \rightarrow \mathbb{Z}_{\square}[-1] = \mathbb{Z}_{\square}[-1]^{\square} = \varinjlim \mathbb{Z}_{\square}[-1, n]$

$$2) \mathbb{Z}_{p, \square} = (\mathbb{Z}_p, \mathbb{Z}_{p, \square}[S] = \mathbb{Z}_p[S]^{\square} = \lim \mathbb{Z}_p[S_i])$$

$$3) A \in \text{Ring} \quad (A, \mathbb{Z})_{\square} = (\underline{A}, S) \mapsto \mathbb{Z}_{\square}[S] \otimes_{\mathbb{Z}} A$$

$$4) A = \text{fn. gen. } \mathbb{Z}\text{-alg}$$

$$A_{\square} = (A, A_{\square}[S] = \lim A[S_i])$$

Def A pre-analytic ring A is analytic if $A[-]$ satisfies (*) in talk 5.

i.e.

$$\forall C \in \text{Cpl}_{\geq 0}(\text{Mod}_A^{\text{word}})$$

$$C_i = \bigoplus_{k \in I_i} A[J_k]$$

$$\dots \rightarrow C_i \rightarrow C_{i-1} \rightarrow \dots \rightarrow C_1 \rightarrow C_0 \rightarrow 0$$

$$\text{RHom}_A(A[S], C) \simeq \text{RHom}_A(\underline{A}[S], C)$$

Prop 7.5 A analytic

Then we have Prop in Abstract framework.

$$\underline{\text{Def}} \quad \text{Mod}_A^{\text{word}} \supset \text{Mod}_A^{\text{word}} = \{ M \in \text{Mod}_A^{\text{word}} \mid \forall S \in \mathbb{P} \}$$

$$\text{Hom}_A(A[S], M) \simeq \mathcal{M}(S)$$

$$\text{Hom}_A(\underline{A}[S], M)$$

- $\text{Mod}_A^{\text{word}}$ is abelian, stable under ...

- $\text{Mod}_A^{\text{cond}}$ is abelian, stable under \dots

- $A[S]$ are compact projective generators of $\text{Mod}_A^{\text{cond}}$

- If \underline{A} comm, sym. mon. $\otimes_{\underline{A}}$

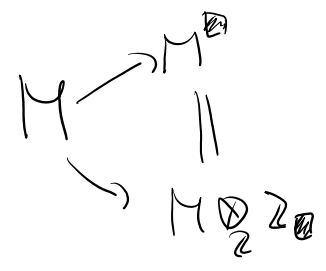
- Similarly in $\mathcal{D}(\text{Mod}_A^{\text{cond}})$

$$\mathcal{D}(A) = \mathcal{D}(\text{Mod}_A^{\text{cond}})$$

Cor $\mathbb{Z}[S]^{\square} = \lim \mathbb{Z}[S_i]$ is solid

$\Rightarrow \mathbb{Z}_{\square} = (\mathbb{Z}, \mathbb{Z}[S]^{\square})$ is analytic

We have $\text{Solid} = \text{Mod}_{\mathbb{Z}_{\square}}^{\text{cond}}$



§ Maps of analytic ngs

Def A, B anal. ngs

A map of anal. ngs $f: A \rightarrow B$ is a map of cond. ngs

$f: \underline{A} \rightarrow \underline{B}$ s.t. $\forall S \in \mathcal{D}$, $B[S]$ as \underline{A} -module
lies in $\text{Mod}_A^{\text{loc}}$

i.e. $\text{Hom}_{\underline{A}}(A[S], B[S]) \cong BS$

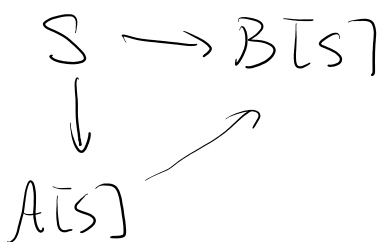


Prop $f: A \rightarrow B$ map of analytic rings

$$\begin{array}{ccc}
 1) \text{ Mod}_A^{\text{loc}} & \xrightarrow{\otimes_A B} & \text{Mod}_B^{\text{loc}} \\
 \downarrow \otimes_A A & & \downarrow \otimes_B B \\
 \text{Mod}_A^{\text{loc}} & \xrightarrow{\otimes_A B} & \text{Mod}_B^{\text{loc}}
 \end{array}$$

2) Similarly on $\mathcal{D}(-)$

$\mathcal{D}/\{A[S] \mid S \in \mathcal{D}\}$ form a family of compact proj. generators
of $\text{Mod}_A^{\text{loc}}$



□

Prop \mathbb{Z}_p & (A, \mathbb{Z}) are analytic rings

D/ Need: $\forall S \in \mathcal{D}$, $\forall C$ good

$$\text{RHom}_A(A[S], C) \xrightarrow{\sim} \text{RHom}_A(\underline{A}[S], C)$$

$$\parallel$$

$$\text{RHom}_A(A[S], C)$$

$$\parallel$$

$$\text{RHom}(\mathbb{Z}[S], C)$$

\parallel

$$\text{RHom}(\mathbb{Z}_p[S], C)$$

\mathbb{Z}_p analytic $\Rightarrow C$ solid

$$(A, \mathbb{Z})_{\mathbb{Z}_p}[S] = \mathbb{Z}_p[S] \otimes_{\mathbb{Z}}^L A$$

$$\otimes_{\mathbb{Z}}^L A \Rightarrow \text{ok.} \quad \square$$

$$(A, A^+) \rightsquigarrow (A, A^+)_{\mathbb{Z}_p} \text{ analytic ring}$$

$$(\mathbb{Q}_p, \mathbb{Z}_p)$$